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Guidance Law Design by Adaptive Fuzzy Sliding-Mode Control

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This paper proposes three fuzzy-logic-based commands to line-of-sight guidance laws. The principle of command to line-of-sight guidance is to force a missile to fly as close as possible along the instantaneous line of sight joining the ground tracker and the target. First, the fuzzy logic control and the fuzzy sliding-mode control guidance laws are presented. In addition, an adaptive fuzzy sliding-mode control design method is developed and is applied for the command to line-of-sight guidance law design. In the design of the adaptive fuzzy sliding-mode control guidance law, the adaptive laws based on a Lyapunov function are developed to adjust the parameters of the fuzzy rules and the uncertainty bound. Thus, the stability of the system can be guaranteed. Finally, two engagement scenarios are examined and a comparison between the proposed fuzzy logic control guidance law, fuzzy sliding-mode control guidance law, adaptive fuzzy sliding-mode control guidance law, and a model-based feedback linearization guidance law is made.

Nomenclature

a_t = target acceleration
a_{ty} = yaw acceleration of target
a_{tz} = pitch acceleration of target
a_x = axial acceleration of missile
a_{yc} = yaw acceleration command
a_{zc} = pitch acceleration command
$c\theta$ = $\cos(\theta)$
D = drag force
g = gravity acceleration
M = mass of missile
$s\theta$ = $\sin(\theta)$
T = thrust force
v_m = missile velocity
v_t = target velocity
(X_I, Y_I, Z_I) = missile inertial frame
(X_L, Y_L, Z_L) = line-of-sight (LOS) frame
(X_M, Y_M, Z_M) = body frame
(x_m, y_m, z_m) = missile position in inertial frame
(x_t, y_t, z_t) = target position in inertial frame
γ_m = true elevation angle of LOS to missile
$\hat{\gamma}_m$ = estimated elevation angle of LOS to missile
γ_t = true elevation angle of LOS to target
$\hat{\gamma}_t$ = estimated elevation angle of LOS to target
θ_m = pitch angle of missile
θ_t = pitch angle of target
σ_m = true azimuth angle of LOS to missile
$\hat{\sigma}_m$ = estimated azimuth angle of LOS to missile
σ_t = true azimuth angle of LOS to target
$\hat{\sigma}_t$ = estimated azimuth angle of LOS to target
ϕ_{mc} = roll angle command
ψ_m = yaw angle of missile
ψ_t = yaw angle of target

I. Introduction

FUZZY logic control (FLC) using linguistic information can model the qualitative aspects of human knowledge and reasoning processes without employing precise quantitative analyses. It also possesses several advantages such as robustness, a model-free, universal approximation theorem, and a rule-based algorithm. Most

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of the operations in an FLC system use the error and change of error as the fuzzy input variables. However, the stability analysis for general FLC systems is still lacking. Recently, some researchers have proposed FLC designs based on the sliding-mode control scheme. This type of controller is referred to as the fuzzy sliding-mode controller (FSMC).[1–3] The sliding-mode control is a control law defined by a rapid switching between two values; with this type of control, the states of the system are attracted to a sliding surface, where the states remain thereafter. For the system to operate in this state of sliding mode, the frequency of switching must be quite high (theoretically infinite). Of course, this description applies to the ideal case, and in practice, because of to the finite switching frequency, the trajectories can only be guaranteed to lie within a small neighborhood of the sliding surface. The trajectories of the system must be directed toward the sliding surface in the immediate vicinity to achieve the local stability in the neighborhood of the sliding surface.[4,5] Because only one variable is defined as the input variable for fuzzy rules, the main advantage of the FSMC is that its number of the fuzzy rules is smaller than that for the FLC. Moreover, by using the sliding-mode control, the system possesses more robustness against parameter variations and external disturbances; and, by using the sliding surface, the FSMC system can be easily designed to guarantee the system's stability in the Lyapunov sense.[3] In the FLC and FSMC designs, the fuzzy rules should be preconstructed to achieve the design performance by trial and error; however, this trial-and-error tuning procedure is time consuming. To tackle this problem, another fuzzy controller design approach is the adaptive fuzzy logic control (AFLC).[6–10] Based on the universal approximation theorem,[6] the AFLC design method can provide a stabilizing controller in the Lyapunov sense even for nonlinear systems with dominant uncertain nonlinearities by using sufficiently complex approximation functions. With this approach, the fuzzy rules can be automatically adjusted to achieve satisfactory system response by an adaptive law.

The principle of command to line of sight (CLOS) guidance is to force a missile to fly as close as possible along the instantaneous line of sight (LOS) joining the ground tracker and the target. Theoretically, the missile–target dynamics equation is nonlinear and time varying, partly because the equations of the motion are best described in an inertial coordinate system, whereas aerodynamic forces and moments are represented in the missile and target body axis system.[11] Many different guidance laws have been developed over the years, and research on improved guidance laws is continuing.[11–15] However, these methods have resulted in complicated controllers, and some of the guidance laws require knowledge of the maneuvering model of the target. In recent years, guidance law designs based on FLC have been presented.[16,17] However, these FLC guidance laws used the error and change of error as the input variables of fuzzy rules, and their fuzzy rules must be preconstructed by human knowledge.

This paper proposes three fuzzy-logic-based CLOS guidance laws referred to as the FLC, FSMC, and adaptive fuzzy sliding-mode control (AFSMC) guidance laws. By defining the sliding surface as the input variable of fuzzy rules, the fuzzy rules of the FSMC and AFSMC guidance laws can be reduced to a minimum. In the design of the FLC and FSMC guidance laws, the fuzzy rules should be preconstructed by human knowledge. However, for the AFSMC guidance law, the adaptive laws are derived to automatically adjust the parameters of the fuzzy rules and the uncertainty bound. The proposed AFSMC has the advantages that it can automatically adjust the fuzzy rules, such as the AFLC, and it can reduce the fuzzy rules, such as the FSMC. These adaptive laws are derived in the Lyapunov sense; thus, the stability of the system can be guaranteed. The proposed FLC, FSMC, and AFSMC guidance laws are derived in model-free conditions; that is, the maneuvering models of the missile and target are not necessary for the derivation of guidance laws. In simulations, the comparison between the proposed FLC, FSMC, and AFSMC guidance laws and a model-based feedback linearization (FBLN)[13] guidance law is examined for two engagement scenarios. Simulation results demonstrate that the proposed FLC, FSMC, and AFSMC guidance laws can handle targets coming from different directions and achieve satisfactory performance. Furthermore, the AFSMC guidance law is found to achieve smaller miss distance than the other guidance laws because the adaptive schemes are applied.

II. Formulation of Missile–Target Engagement

To describe the missile–target engagement, an LOS coordinate is chosen as the reference coordinate system. Figure 1 depicts the three-dimensional pursuit situation. The origin of the inertial frame is located at the ground tracker. The roll angle of the missile is assumed to be controlled to zero by the missile's attitude control system, and both the elevation loop and the azimuth loop are assumed to be decoupled. The guided missiles considered here belong to the skid-to-turn category, which is composed of two lateral acceleration commands: yaw (a_{yc}) and pitch (a_{zc}). Defining the LOS frame as depicted in Fig. 2, the three-dimensional CLOS guidance problem can be converted to a tracking problem. The CLOS guidance involves guiding the missile onto the LOS to target. Therefore, a reasonable choice of tracking error can be

$$\delta \triangleq \begin{bmatrix} \Delta\sigma \\ \Delta\gamma \end{bmatrix} = \begin{bmatrix} \hat{\sigma}_t - \hat{\sigma}_m \\ \hat{\gamma}_t - \hat{\gamma}_m \end{bmatrix} \tag{1}$$

The design problem involves designing a controller that controls the signal $\boldsymbol{u} \triangleq [a_{yc}, a_{zc}]^T$ to drive $[\Delta\sigma, \Delta\gamma]^T$ to zero. The same design algorithm will be applied for azimuth and elevation angle control. In the following, the azimuth angle control is chosen as an example.

Assume that $e(t) = \Delta\sigma$ represents the azimuth loop tracking error, $R(t)$ represents the tracker-to-missile range, and $R_y(t)$ represents the component of $R(t)$ in axis Y_L. Because $e(t)$ is a small variable, it can be obtained as[18]

$$e(t) = R_y(t)/R(t) \tag{2}$$

Differentiating Eq. (2) twice with respect to time produces

$$\ddot{e}(t) = -a_1(t)e(t) - a_2(t)\dot{e}(t) + b(t)\ddot{R}_y(t) \tag{3}$$

where $a_1(t) = \ddot{R}(t)/R(t)$, $a_2(t) = 2\dot{R}(t)/\ddot{R}(t)$, $b(t) = 1/R(t)$, and $\ddot{R}_y(t) = -a_m(t) + a_t(t)$. In Eq. (3), $a_m(t)$ and $a_t(t)$ represent the missile's acceleration and the target's acceleration in axis Y_L, respectively.

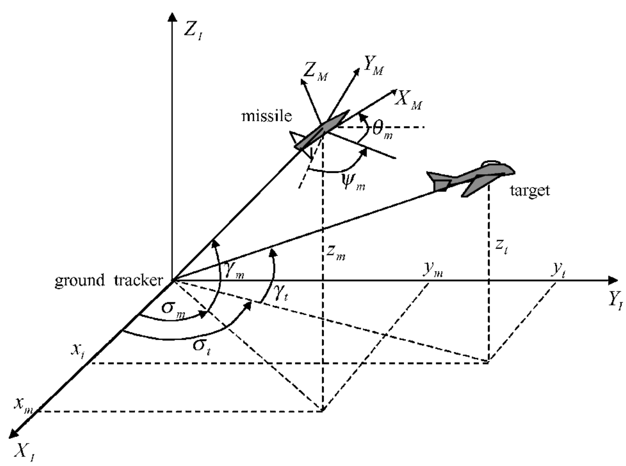

Fig. 1 Three-dimensional pursuit situation.

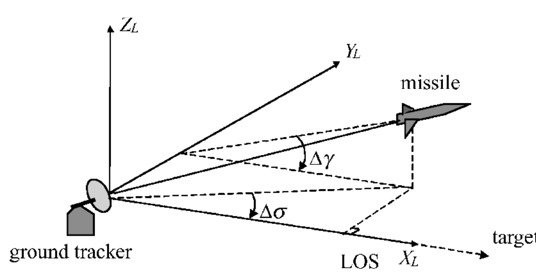

Fig. 2 Definition of tracking error.

Equation (3) can be rewritten as

$$\ddot{e}(t) = -a_1(t)e(t) - a_2(t)\dot{e}(t) + b(t)[-a_m(t) + a_t(t)]$$
$$= F[e(t), \dot{e}(t)] + G(t)[u(t) - d(t)]$$
$$= F(\boldsymbol{e}; t) + G(t)[u(t) - d(t)] \tag{4}$$

where $\boldsymbol{e} = [e(t), \dot{e}(t)]^T$, $F(\boldsymbol{e}; t) = -a_1(t)e(t) - a_2(t)\dot{e}(t)$, $G(t) = -b(t)$, $u(t) = a_m(t)$, and $d(t) = a_t(t)$. In Eq. (4), $u(t)$ and $d(t)$ are the control variable and the disturbance, respectively. In addition, $G(t) = -1/R(t)$ is a negative function and $\dot{G}(t)$ is a positive function. The CLOS guidance involves designing a controller that controls the signal $u(t)$ to drive $e(t)$ to zero when the system is subjected to time-varying parameter variations and unknown target maneuvering disturbances.

III. Fuzzy Logic Controller and Fuzzy Sliding-Mode Controller Design

A. Fuzzy Logic Controller

The FLC CLOS guidance system is depicted in Fig. 3, and the fuzzy control rules are given in the following form:

$$\text{Rule } i: \text{if } e \text{ is } F_e^i \text{ and } \dot{e} \text{ is } F_{\dot{e}}^i, \text{ then } u \text{ is } \rho_i \tag{5}$$

where $\rho_i, i = 1, 2, \ldots, n$ are the singleton control actions and F_e^i and $F_{\dot{e}}^i$ are the labels of the fuzzy sets. The defuzzification of the controller output is accomplished by the method of center of gravity:

$$u_{\text{flc}}[e(t), \dot{e}(t), \rho_i] = \frac{\sum_{i=1}^{n} v_i \times \rho_i}{\sum_{i=1}^{n} v_i} \tag{6}$$

where v_i is the firing weight of the ith rule. The fuzzy rules in Eq. (5) can be constructed by the sense that $e(t)$ and $\dot{e}(t)$ will approach to zero with fast rise time and without large overshoot.

B. Fuzzy Sliding-Mode Controller

The sliding surface plays a very important role in the design of FSMC.[1–3] It can dominate the dynamic behavior of the control system as well as reduce the size of the fuzzy rule base. Besides, the FSMC possesses more robustness against parameter variations and external disturbances than the FLC. Also, by using the sliding surface, the FSMC system can be easily designed to guarantee the system's stability in the Lyapunov sense. A sliding surface is defined as

$$s(t) = \dot{e}(t) + k_1 e(t) + k_2 \int_0^t e(\tau)\,\mathrm{d}\tau \tag{7}$$

where k_1 and k_2 are constant gains. The FSMC CLOS guidance system is depicted in Fig. 4, and the fuzzy control rules are given in the following form:

$$\text{Rule } j: \text{if } s \text{ is } F_s^j, \text{ then } u \text{ is } \alpha_j \tag{8}$$

where $\alpha_j, j = 1, 2, \ldots, m$ are the singleton control actions and F_s^j is the label of the fuzzy set. The defuzzification of the controller output is also accomplished by the method of center of gravity:

$$u_{\text{fz}}(s) = \frac{\sum_{j=1}^{m} w_j \times \alpha_j}{\sum_{j=1}^{m} w_j} \tag{9}$$

where w_j is the firing weight of the jth rule. For the FSMC guidance law design, the fuzzy rules in Eq. (8) can be constructed using the basic idea that if the state is far away from the sliding surface, then a large control effort should be applied, and if the state is near the sliding surface, then a small control effort should be applied, so that the state can quickly reach the sliding surface without large overshoot.

IV. Adaptive Fuzzy Sliding-Mode Controller Design

The AFSMC guidance system is depicted in Fig. 5. If $F(\boldsymbol{e}; t)$, $G(t)$, and $d(t)$ in Eq. (4) are accurately known, the ideal controller can be obtained by

$$u^*(t) = -G(t)^{-1}[F(\boldsymbol{e}; t) + k_1\dot{e}(t) + k_2 e(t) - G(t)d(t)] \tag{10}$$

Substituting Eq. (10) into Eq. (4) gives

$$\ddot{e}(t) + k_1\dot{e}(t) + k_2 e(t) = 0 \tag{11}$$

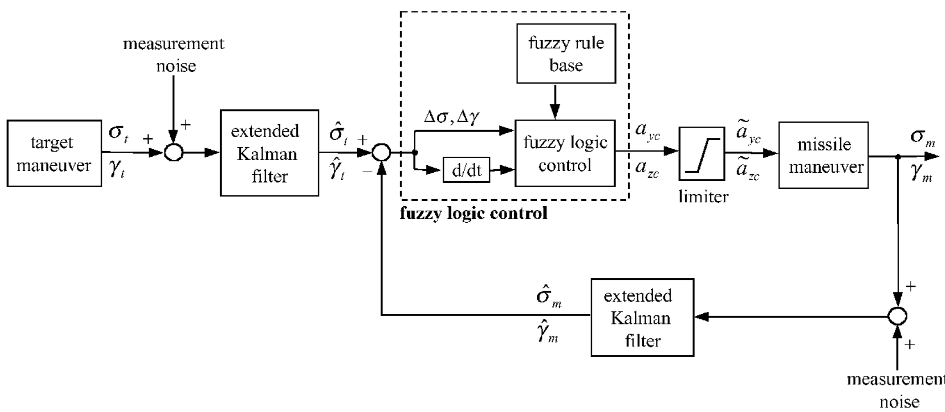

Fig. 3 Fuzzy logic control guidance law design concept.

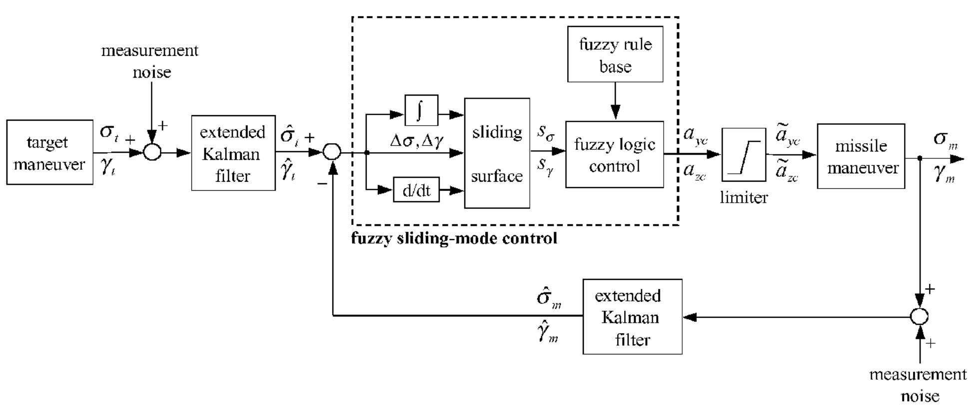

Fig. 4 Fuzzy sliding-mode control guidance law design concept.

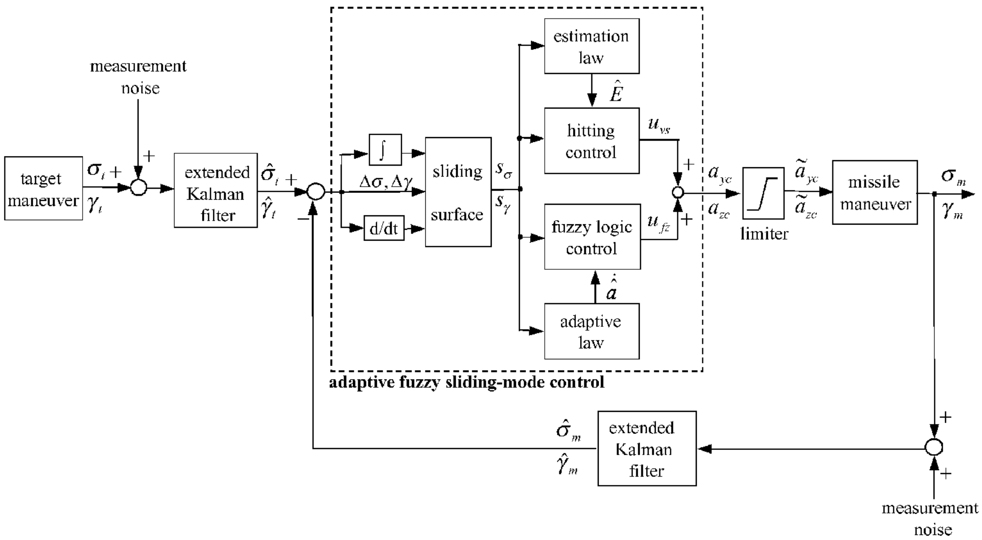

Fig. 5 Adaptive fuzzy sliding-mode control guidance law design concept.

The constant gains k_1 and k_2 can be chosen to correspond to the coefficients of a Hurwitz polynomial that implies that $\lim_{t\to\infty} e(t) = 0$. Since $F(\boldsymbol{e}; t)$, $G(t)$, and $d(t)$ are either time varying or unknown, the ideal controller $u^*(t)$ cannot be implemented. Hence, an AFSMC is designed to approximate this ideal controller.

If α_j is chosen as an adjustable parameter, then Eq. (9) can be rewritten as

$$u_{\text{fz}}[s(t), \boldsymbol{\alpha}] = \boldsymbol{\alpha}^T \boldsymbol{\xi} \tag{12}$$

where $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \ldots, \alpha_m]^T$ is a parameter vector and $\boldsymbol{\xi} = [\xi_1, \xi_2, \ldots, \xi_m]^T$ is a regressive vector with ξ_j defined as

$$\xi_j = \frac{w_j}{\sum_{j=1}^{m} w_j} \tag{13}$$

By the universal approximation theorem,[6] there exists an optimal fuzzy logic system $u^*_{\text{fz}}[s(t), \boldsymbol{\alpha}^*]$ in the form of Eq. (12) such that

$$u^*_{\text{fz}}[s(t), \boldsymbol{\alpha}^*] = \boldsymbol{\alpha}^{*T} \boldsymbol{\xi} \tag{14}$$

where the time invariant parameter vector $\boldsymbol{\alpha}^*$ is defined as

$$\boldsymbol{\alpha}^* = \arg \min_{|a| \le M_\alpha} \left\{ \sup_{|s| \le M_s} [u_{\text{fz}}(s, \boldsymbol{\alpha}) - u^*(t)] \right\} \text{ for all } t \tag{15}$$

and M_s and M_α are specified by the designer. The minimum approximation error is defined as

$$\varepsilon(t) \equiv u^*_{\text{fz}} - u^*, \qquad 0 \le |\varepsilon(t)| \le E \tag{16}$$

where the uncertainty bound E is a positive constant. This uncertainty bound cannot be measured for practical applications. Therefore, a bound estimation is developed to observe the bound of approximation error. Define the estimated error

$$\tilde{E}(t) = \hat{E}(t) - E \tag{17}$$

where $\hat{E}(t)$ is the estimated uncertainty bound.

The control law for the AFSMC system is assumed to take the following form:

$$u[s(t), \hat{\boldsymbol{\alpha}}(t)] = u_{\text{fz}}[s(t), \hat{\boldsymbol{\alpha}}(t)] + u_{\text{vs}}[s(t)] \tag{18}$$

where u_{fz} is the main tracking control, and the hitting control u_{vs} is designed to stabilize the states of the control system around a preselected uncertainty bound. The adaptive laws will be developed to adjust the parameters $\hat{\boldsymbol{\alpha}}(t)$ and $\hat{E}(t)$ to estimate $\boldsymbol{\alpha}^*$ and E, respectively; and the estimation error of fuzzy control effort is denoted as

$$\tilde{u}_{\text{fz}} = u_{\text{fz}} - u^*_{\text{fz}} = \tilde{\boldsymbol{\alpha}}^T \boldsymbol{\xi} \tag{19}$$

where $\tilde{\boldsymbol{\alpha}}(t) = \hat{\boldsymbol{\alpha}}(t) - \boldsymbol{\alpha}^*$.

Substituting Eq. (18) into Eq. (4), it is revealed that

$$\ddot{e}(t) = F(\boldsymbol{e}; t) + G(t)[u_{\text{fz}} + u_{\text{vs}} - d(t)] \tag{20}$$

From Eqs. (10) and (20), the error equation governing the closed-loop system can be obtained as

$$G(t)(u_{\text{fz}} + u_{\text{vs}} - u^*) = \ddot{e}(t) + k_1\dot{e}(t) + k_2 e(t) = \dot{s}(t) \tag{21}$$

Because $G(t) = -1/R(t)$ is negative, the Lyapunov function candidate can be defined as

$$V[s(t), \tilde{\boldsymbol{\alpha}}(t), \tilde{E}(t)] = \frac{1}{2}s^2(t) - \frac{G(t)}{2\eta_1}\tilde{\boldsymbol{\alpha}}^T\tilde{\boldsymbol{\alpha}} - \frac{G(t)}{2\eta_2}\tilde{E}^2 \tag{22}$$

where η_1 and η_2 are positive constants. Differentiating Eq. (22) with respect to time and using Eqs. (17), (19), and (21), it is obtained that

$$\begin{aligned}
\dot{V}[s(t), \tilde{\boldsymbol{\alpha}}(t), \tilde{E}(t)] &= s(t)\dot{s}(t) - \frac{G(t)}{\eta_1}\tilde{\boldsymbol{\alpha}}^T\dot{\tilde{\boldsymbol{\alpha}}} - \frac{\dot{G}(t)}{2\eta_1}\tilde{\boldsymbol{\alpha}}^T\tilde{\boldsymbol{\alpha}} \\
&- \frac{G(t)}{\eta_2}\tilde{E}^T\dot{\tilde{E}} - \frac{\dot{G}(t)}{2\eta_2}\tilde{E}^2 = s(t)G(t)\big(u_{\text{fz}} + u_{\text{vs}} - u^*\big) \\
&- \frac{G(t)}{\eta_1}\tilde{\boldsymbol{\alpha}}^T\dot{\tilde{\boldsymbol{\alpha}}} - \frac{G(t)}{\eta_2}\tilde{E}^T\dot{\tilde{E}} - \frac{\dot{G}(t)}{2\eta_1}\tilde{\boldsymbol{\alpha}}^T\tilde{\boldsymbol{\alpha}} - \frac{\dot{G}(t)}{2\eta_2}\tilde{E}^2 \\
&\le s(t)G(t)\big(u_{\text{fz}} + u_{\text{vs}} - u^* + u^*_{\text{fz}} - u^*_{\text{fz}}\big) - \frac{G(t)}{\eta_1}\tilde{\boldsymbol{\alpha}}^T\dot{\tilde{\boldsymbol{\alpha}}} \\
&- \frac{G(t)}{\eta_2}\tilde{E}\dot{\tilde{E}} = s(t)G(t)\big[\tilde{\boldsymbol{\alpha}}^T\boldsymbol{\xi} + u_{\text{vs}} + \varepsilon(t)\big] - \frac{G(t)}{\eta_1}\tilde{\boldsymbol{\alpha}}^T\dot{\tilde{\boldsymbol{\alpha}}} \\
&- \frac{G(t)}{\eta_2}\tilde{E}\dot{\tilde{E}} = \tilde{\boldsymbol{\alpha}}^T G(t)\left[s(t)\boldsymbol{\xi} - \frac{\dot{\tilde{\boldsymbol{\alpha}}}}{\eta_1}\right] \\
&+ s(t)G(t)[u_{\text{vs}} + \varepsilon(t)] - \frac{G(t)}{\eta_2}\tilde{E}\dot{\tilde{E}}
\end{aligned} \tag{23}$$

If the adaptive laws and the hitting control are chosen as

$$\dot{\hat{\boldsymbol{\alpha}}} = \dot{\tilde{\boldsymbol{\alpha}}} = \eta_1 s(t)\boldsymbol{\xi} \tag{24}$$

$$\dot{\hat{E}} = \dot{\tilde{E}} = -\eta_2|s(t)|\text{sgn}[G(t)] = \eta_2|s(t)| \tag{25}$$

$$u_{\mathrm{vs}}(s) = -\hat{E}\,\mathrm{sgn}[s(t)G(t)] = \hat{E}\,\mathrm{sgn}[s(t)] \tag{26}$$

then inequality (23) becomes

$$\begin{aligned}\dot{V}[s(t), \tilde{\alpha}(t), \tilde{E}(t)] &\le \varepsilon(t)s(t)G(t) - \hat{E}|s(t)G(t)| \\ &\quad + \tilde{E}|s(t)G(t)| \le |\varepsilon(t)||s(t)G(t)| - E|s(t)G(t)| \\ &= -[E - |\varepsilon(t)|]|s(t)G(t)| \le 0 \end{aligned} \tag{27}$$

where $\mathrm{sgn}(\cdot)$ is a sign function. This implies that $\dot{V}[s(t), \tilde{\alpha}(t), \tilde{E}(t)]$ is negative semidefinite.

In summary, the AFSMC guidance law is presented in Eq. (18), where u_{fz} is given in Eq. (12) with the parameters $\hat{\alpha}$ adjusted by Eq. (24) and where u_{vs} is given in Eq. (26) with the parameter $\hat{E}$ adjusted by Eq. (25). By applying this guidance law, the AFSMC guidance system can be guaranteed to be stable.

V. Simulation Results

For simulation, the motion of the missile in the inertial frame can be represented by[13]

$$\begin{aligned} \ddot{x}_m &= a_x c\theta_m c\psi_m - \tilde{a}_{\mathrm{yc}}(s\phi_{\mathrm{mc}} s\theta_m c\psi_m + c\phi_{\mathrm{mc}} s\psi_m) \\ &\quad - \tilde{a}_{\mathrm{zc}}(c\phi_{\mathrm{mc}} s\theta_m c\psi_m - s\phi_{\mathrm{mc}} s\psi_m) \\ \ddot{y}_m &= a_x c\theta_m s\psi_m - \tilde{a}_{\mathrm{yc}}(s\phi_{\mathrm{mc}} s\theta_m s\psi_m - c\phi_{\mathrm{mc}} c\psi_m) \\ &\quad - \tilde{a}_{\mathrm{zc}}(c\phi_{\mathrm{mc}} s\theta_m s\psi_m + s\phi_{\mathrm{mc}} c\psi_m) \\ \ddot{z}_m &= a_x s\theta_m + \tilde{a}_{\mathrm{yc}} s\phi_{\mathrm{mc}} c\theta_m + \tilde{a}_{\mathrm{zc}} c\phi_{\mathrm{mc}} c\theta_m - g \\ \dot{\psi}_m &= \tilde{a}_{\mathrm{yc}} c\phi_{\mathrm{mc}}/(v_m c\theta_m) - \tilde{a}_{\mathrm{zc}} s\phi_{\mathrm{mc}}/(v_m c\theta_m) \\ \dot{\theta}_m &= \tilde{a}_{\mathrm{yc}} s\phi_{\mathrm{mc}}/v_m + \tilde{a}_{\mathrm{zc}} c\phi_{\mathrm{mc}}/v_m - g c\theta_m/v_m \end{aligned} \tag{28}$$

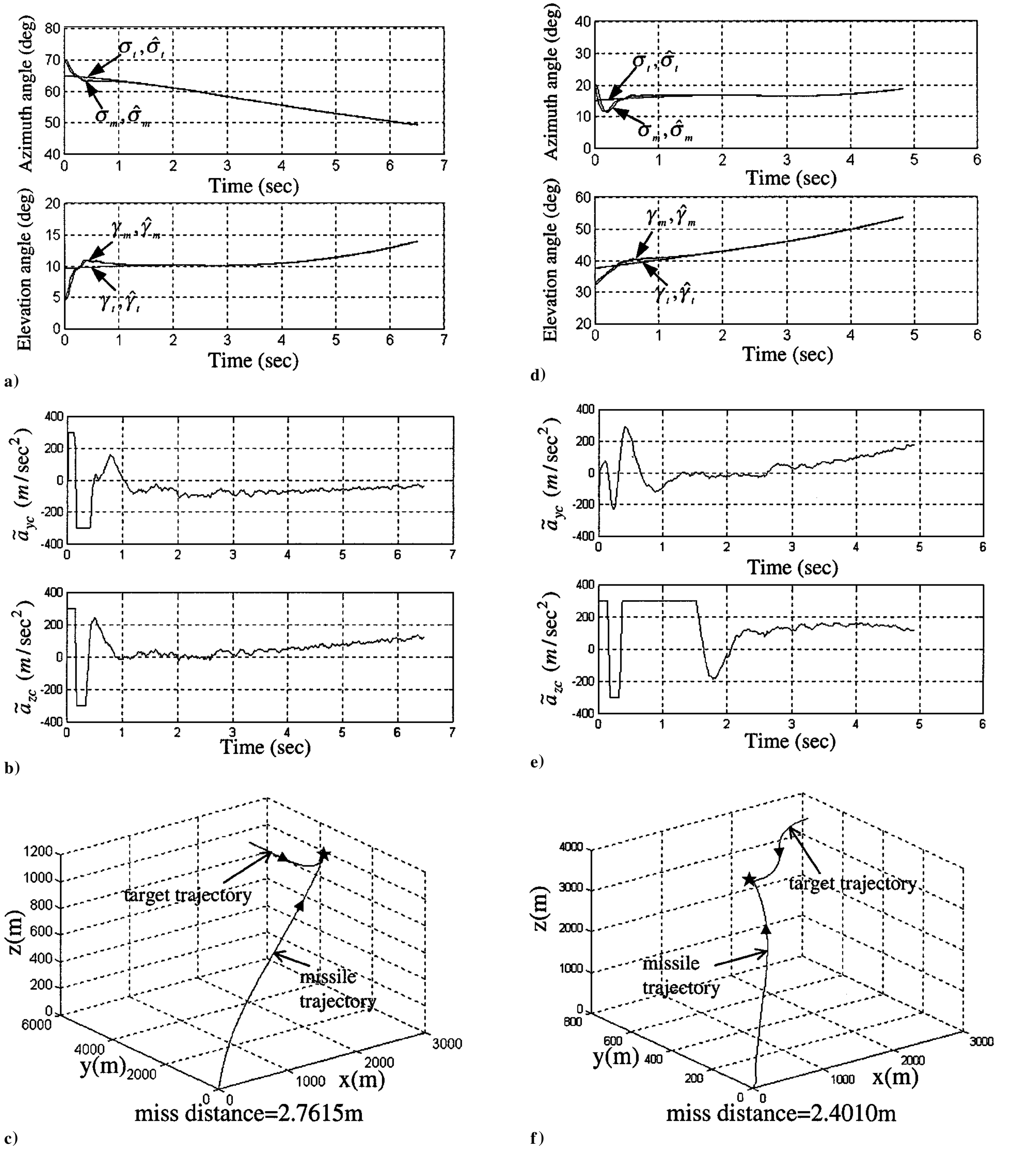

Fig. 6 Engagement responses of feedback linearization guidance law.

where v_m denotes the velocity of the missile given by

$$v_m \triangleq \left(\dot{x}_m^2 + \dot{y}_m^2 + \dot{z}_m^2\right)^{\frac{1}{2}} \tag{29}$$

and a_x represents the axial acceleration of the missile given by

$$a_x \triangleq (T - D)/M \tag{30}$$

The simplified dynamics of target motion can be represented in the inertial frame as follows:

$$\ddot{x}_t = -a_{ty} s\psi_t - a_{tz} s\theta_t c\psi_t, \qquad \ddot{y}_t = a_{ty} c\psi_t - a_{tz} s\theta_t s\psi_t$$

$$\ddot{z}_t = a_{tz} c\theta_t - g, \qquad \dot{\psi}_t = a_{ty}/(v_t c\theta_t), \qquad \dot{\theta}_t = (a_{tz} - gc\theta_t)/v_t \tag{31}$$

where v_t is given by

$$v_t \triangleq \left(\dot{x}_t^2 + \dot{y}_t^2 + \dot{z}_t^2\right)^{\frac{1}{2}} \tag{32}$$

However, the parameters in Eq. (31) may be time varying and even unknown, because the flight direction, velocity, and acceleration of the target may be changeable and unknown in a practical engagement scenario. The uncertainty in the target's motion is traditionally handled using a stochastic model. If the stochastic model is lacking (that is, it does not adequately represent all of the possible random, highly dynamic maneuvers that a modern tactical aircraft can make), then there will be scenarios where the guidance law yields poor intercept performance. An alternative is to forgo this difficult modeling task and treat the target's maneuvering as a disturbance. It should be emphasized that the derivation of fuzzy-logic-based CLOS

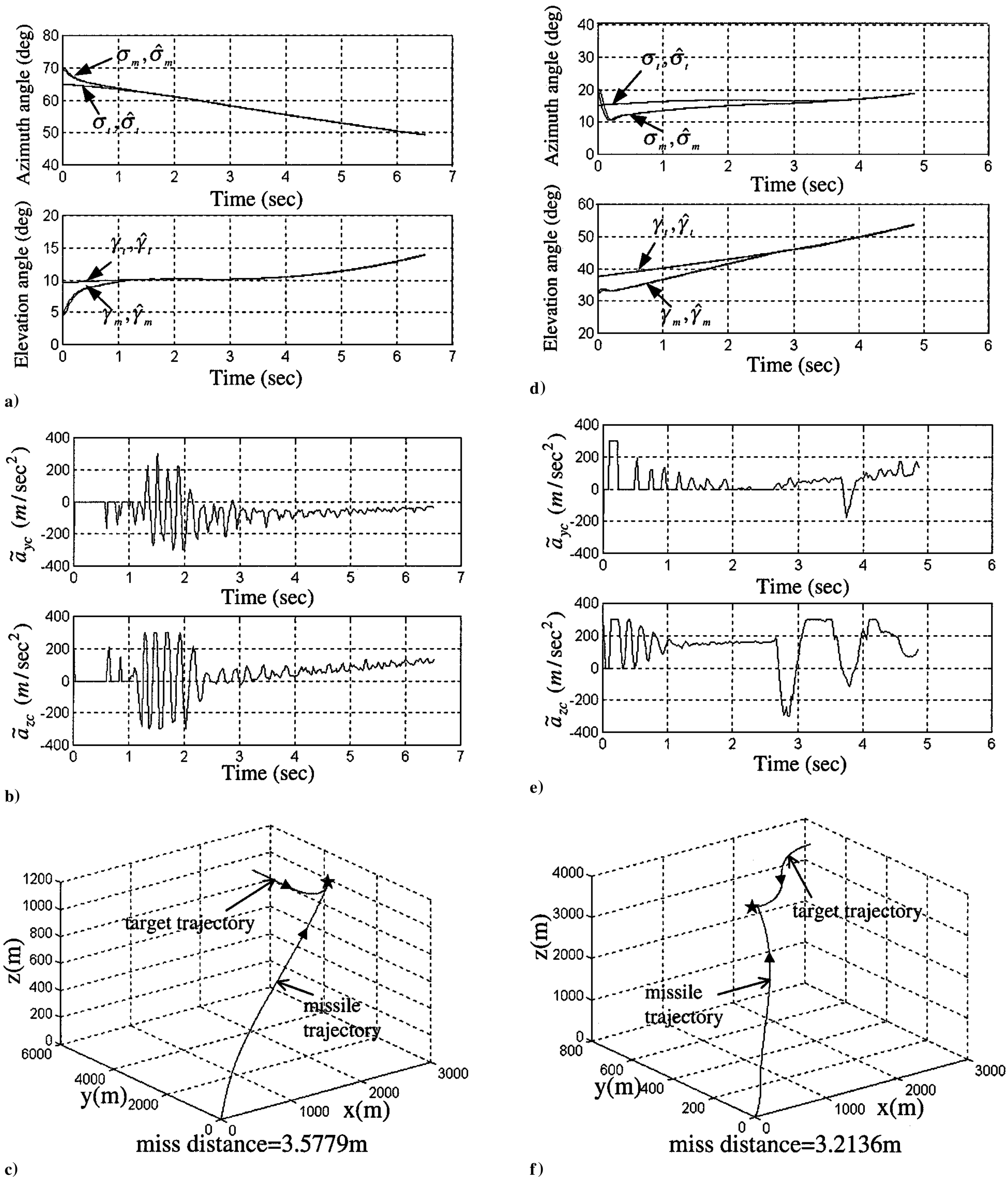

Fig. 7 Engagement responses of fuzzy logic control guidance law.

guidance laws do not need to use the missile model in Eq. (28) and target model in Eq. (31). These models are used only for simulations and for the FBLN guidance law. To justify the merit of the proposed guidance laws, a comparison between the proposed guidance laws with a model-based feedback linearization guidance law[13] is made. In the simulations, measurement random noises with magnitude ±0.1 deg are added to the LOS angle of the missile and target, and an extended Kalman filter is used to filter these random noises. For azimuth-loop control, the control law u in Eq. (18) is denoted as a_{yc}, and the elevation-loop control law u is denoted as a_{zc}. A 30 g ($g = 9.8\ \mathrm{m/s^2}$) maneuvering limiter is included to present the limitation of missile's maneuverability. Thus, the acceleration commands are expressed as

$$\tilde{a}_{yc} = \mathrm{sat}(a_{yc}, 30\,g) \tag{33}$$

$$\tilde{a}_{zc} = \mathrm{sat}(a_{zc}, 30\,g) \tag{34}$$

where

$$\mathrm{sat}(a, b) = \begin{cases} a & \text{for} \quad a \le |b| \\ b \cdot \mathrm{sgn}(a) & \text{for} \quad a > |b| \end{cases} \tag{35}$$

and $\tilde{a}_{yc}$ and $\tilde{a}_{zc}$ denote acceleration commands for the azimuth loop and the elevation loop, respectively.

Two simulation scenarios are examined. Tables 1 and 2 list the detailed data used for the simulations. Assume that the target maneuvers with $a_{ty} = 5\,g$ and $a_{tz} = -g$ for the first 2.5 seconds and then with $a_{ty} = -5\,g$ and $a_{tz} = 5\,g$ until interception. The performance evaluations of the designed guidance systems consist of miss distance and the responses of tracking errors. For the feedback linearization guidance law,[13] the simulation results are depicted in Figs. 6a–6c for scenario 1, and Figs. 6d–6f for scenario 2, respectively. For the FLC guidance law, the fuzzy rules are given in Table 3, in which the fuzzy labels used are negative

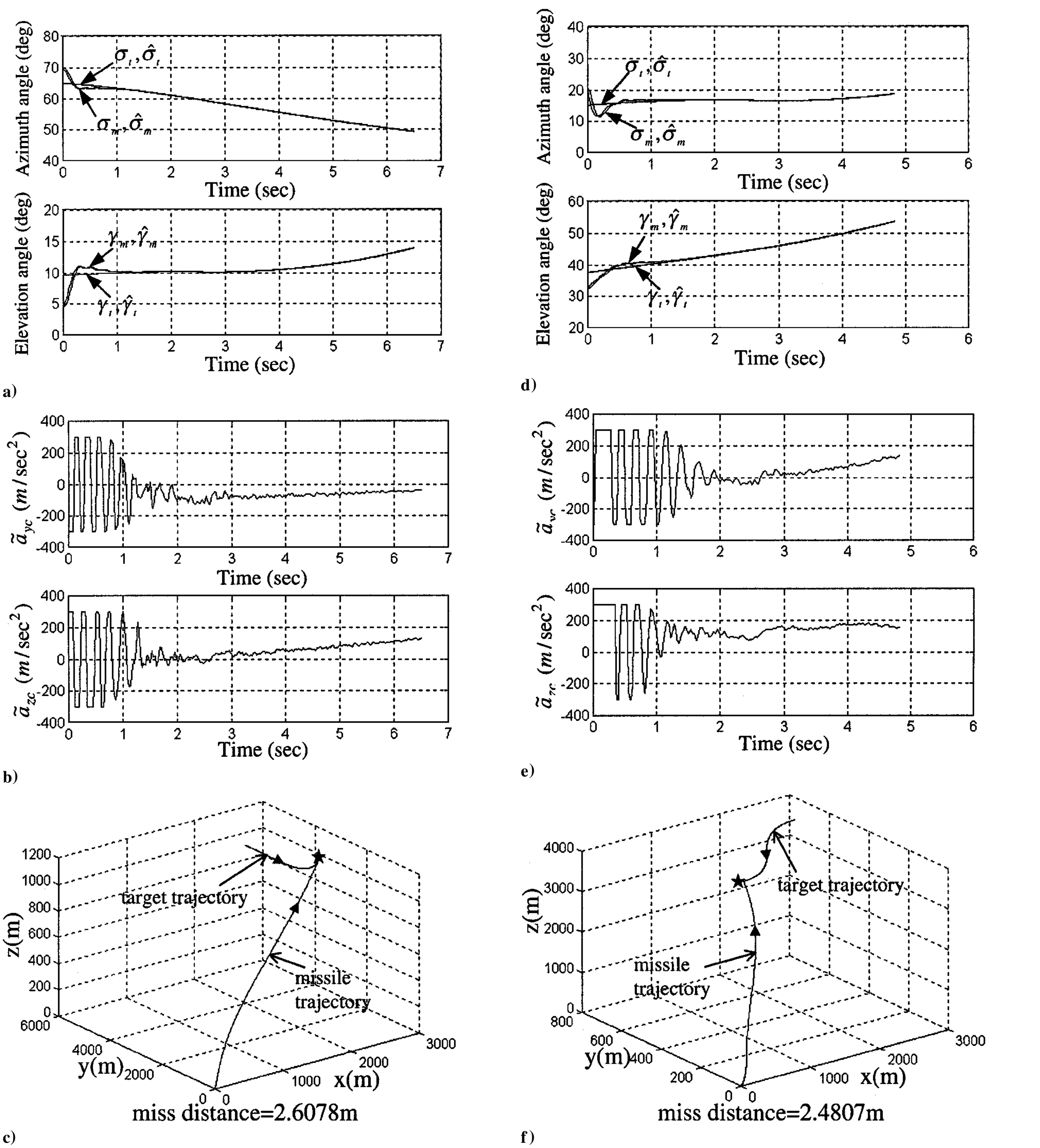

Fig. 8 Engagement responses of fuzzy sliding-mode control guidance law.

Table 1 Initial data used for simulations

State	Scenario 1	Scenario 2
$x_t(0), y_t(0), z_t(0)$ m	2500, 5361.9, 1000	3500, 950, 2800
$\dot{x}_t(0), \dot{y}_t(0), \dot{z}_t(0)$ m/s	0, −340, 0	−340, 0, 0
$\psi_t(0), \theta_t(0)$ deg	−90, 0	180, 0
$x_m(0), y_m(0), z_m(0)$ m	14.32, 39.34, 3.36	14.65, 5.43, 10.01
$\dot{x}_m(0), \dot{y}_m(0), \dot{z}_m(0)$ m/s	70.84, 151.92, 28.32	129.65, 12.87, 92.42
$\psi_m(0), \theta_m(0)$ deg	65, 9.59	20.34, 32.65
$\Delta\sigma(0), \Delta\gamma(0)$ deg	−5, 5	−5, 5

Table 2 Parameter data used for simulations

Parameter	Value
$(T - D)/M$	$\begin{cases} 340 \text{ m/s}^2 & 0 < t < 2 \\ -44.1 \text{ m/s}^2 & t > 2 \end{cases}$
ϕ_{mc}	0 deg
Guidance command frequency	50 Hz

big (NB), negative medium (NM), negative small (NS), zero (ZO), positive small (PS), positive medium (PM), and positive big (PB), where the membership functions of the fuzzy sets are given in a triangular form. The simulation results for the FLC guidance law are depicted in Figs. 7a–7c for scenario 1, and Figs. 7d–7f for scenario 2, respectively. For the FSMC guidance law, the fuzzy rules are $S = \alpha_j$, NB = −1.0000, NM = −0.8133, NS = −0.4287, ZO = 0.0000, PS = 0.4287, PM = 0.8133, PB = 1.0000, and the

Table 3 Fuzzy rules for fuzzy logic control guidance law

$\dot{e}\backslash e$	NB	NM	NS	ZO	PS	PM	PB
NB	−1.0000	−1.0000	−1.0000	−1.0000	−0.8133	−0.4287	0.0000
NM	−1.0000	−1.0000	−1.0000	−0.8133	−0.4287	0.0000	0.4287
NS	−1.0000	−1.0000	−0.8133	−0.4287	0.0000	0.4287	0.8133
ZO	−1.0000	−0.8133	−0.4287	0.0000	0.4287	0.8133	1.0000
PS	−0.8133	−0.4287	0.0000	0.4287	0.8133	1.0000	1.0000
PM	−0.4287	0.0000	0.4287	0.8133	1.0000	1.0000	1.0000
PB	0.0000	0.4287	0.8133	1.0000	1.0000	1.0000	1.0000

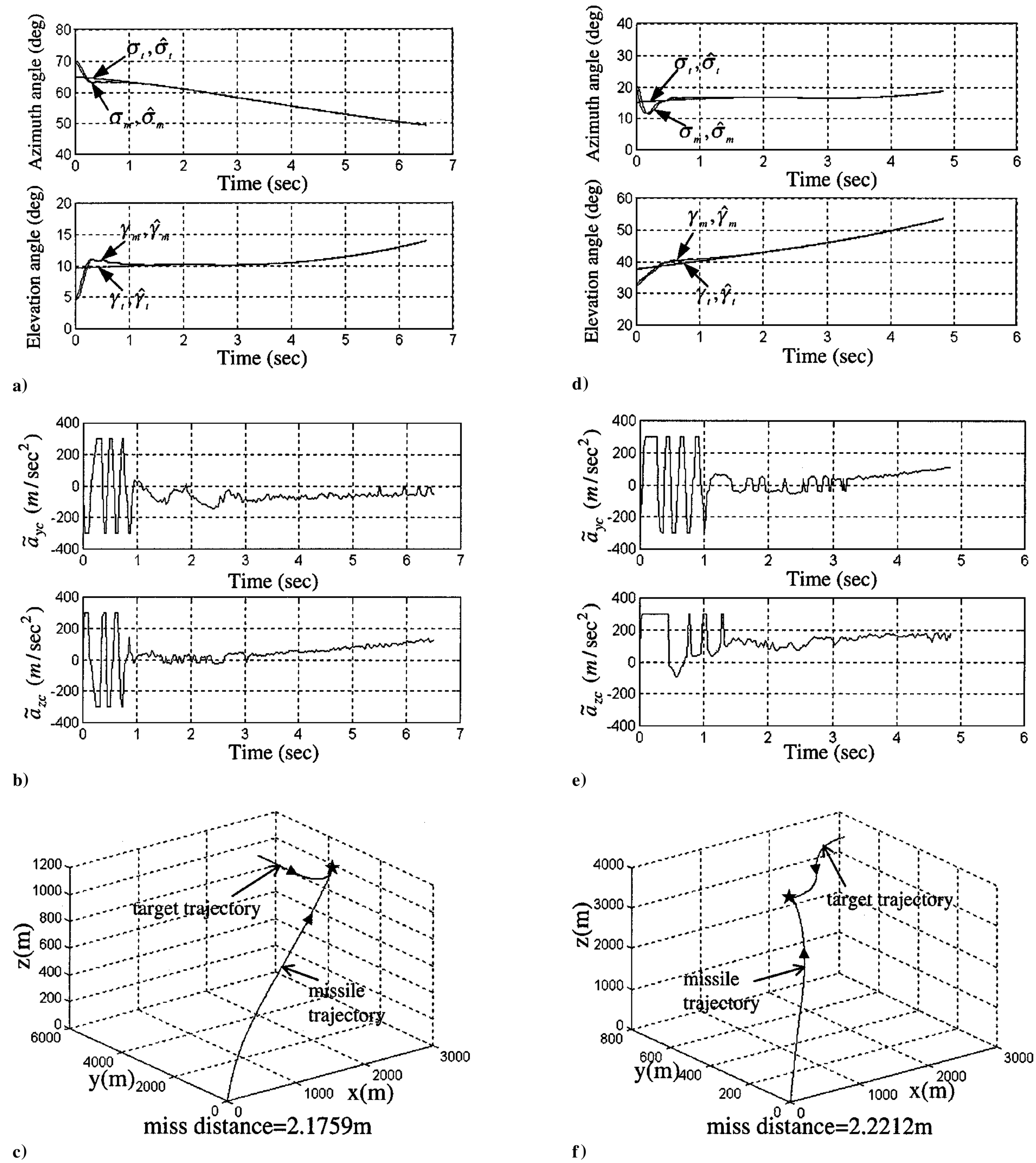

Fig. 9 Engagement responses of adaptive fuzzy sliding-mode control guidance law.

Table 4 Miss distances (m) for FBLN, FLC, FSMC, and AFSMC guidance laws

Design method	Scenario 1	Scenario 2
FBLN	2.7615	2.4010
FLC	3.5779	3.2136
FSMC	2.6078	2.4807
AFSMC	2.1759	2.2212

membership functions of the fuzzy sets are also given in a triangular form. The simulation results for the FSMC guidance law are depicted in Figs. 8a–8c for scenario 1, and Figs. 8d–8f for scenario 2, respectively. In the proposed AFSMC guidance law, the singleton control actions are initiated from zero and are learned from the developed adaptive law with $\eta_1 = 0.3$ and $\eta_2 = 0.003$. The simulation results for the AFSMC guidance law are depicted in Figs. 9a–9c for scenario 1, and Figs. 9d–9f for scenario 2, respectively. The simulation results show that the extended Kalman filter has been tuned and has been providing good estimates of the true LOS angles. The comparison of the simulation results is summarized in Table 4, which shows that the AFSMC guidance law can achieve a smaller miss distance than the other guidance laws. Moreover, for the AFSMC, the online estimation of the uncertainty bound E can obtain a small value of $\hat{E}$ as the fuzzy controller approaches the ideal controller. This estimated uncertainty bound $\hat{E}$ is generally smaller than a preselected uncertainty bound used in the sliding-mode control for coping with the uncertainties and disturbances. Therefore, the chattering phenomenon of the control effort can be reduced in the AFSMC design method.

VI. Conclusions

The FLC, FSMC, and AFSMC command to LOS guidance laws are proposed. The fuzzy rules of FLC and FSMC should be preconstructed by trial-and-error tuning. By the AFSMC, the fuzzy rules can be learned online by an adaptive law, and the stability of the proposed AFSMC guidance system can be guaranteed. A comparison of the FLC, FSMC, AFSMC, and FBLN guidance laws for two different engagement scenarios is made. Simulation results demonstrate that the proposed FLC, FSMC, and AFSMC guidance laws can achieve satisfactory performance for different engagement scenarios. Furthermore, the AFSMC guidance law is found to achieve smaller miss distance than the FLC and FSMC guidance laws because the adaptive schemes are applied. Comparison between the AFSMC and FBLN guidance laws also shows that the AFSMC guidance law can achieve smaller miss distance and reduce implementation complexity while paying the price of a little chattering in the control effort.

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